NUWC-NL Technical Report 10,131
1 September 1992

A Comparison of Acoustic Impedance Measurement Techniques

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PREFACE

The research described in this report was prepared under the *Acoustic Array Technology Program* as part of the Submarine/Surface Ship USW Surveillance Program sponsored by the Antisubmarine Warfare/Undersea Technology Directorate of the Office of Naval Technology: Program Element 0602314N; ONT Block Program UN3B; Project No. RJ14R13; NUWC Job Order No. V60010; NUWC Principal Investigator, D.A. Hurdis (Code 2141); Program Director G.C. Connolly (Code 2192). The sponsoring activity's Technology Area Manager for Undersea Target Surveillance is T.G. Goldsberry (ONT 231).

The author of this report is located at the Naval Undersea Warfare Center, New London Detachment, New London, CT 06320. The technical reviewer for this report was W.L. Keith (Code 2141)

The author wishes to thank the Dynamics Systems and Controls Laboratory of Michigan State University, East Lansing, Michigan for the use of their impedance tube.

REVIEWED AND APPROVED: 1 SEPTEMBER 1992

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REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
1 September 1992

3. REPORT TYPE AND DATES COVERED
Progress

4. TITLE AND SUBTITLE
A Comparison of Acoustic Impedance Measurement Techniques

5. FUNDING NUMBERS
PE 62314N
PR RJ14R13

6. AUTHOR(S)
Andrew J. Hull

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Undersea Warfare Center Detachment
New London, CT 06320

8. PERFORMING ORGANIZATION REPORT NUMBER
TR 10,131

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Technology
Office of the Chief of Naval Research
Arlington, VA 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

This report describes an investigation that compares two acoustic impedance measurement techniques. The first is a wave decomposition method that breaks the wave energy into reflected and incident components and then calculates acoustic impedance. The second is an inverse eigenvalue method that inverts the functional form of impedance tube eigenvalues and then calculates acoustic impedance. An experiment comparing the measured acoustic impedance from both methods results in similar values.

14. SUBJECT TERMS
Acoustic Impedance
Eigenvalue Method
Inverse Problem
Wave Decomposition

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

A COMPARISON OF ACOUSTIC IMPEDANCE MEASUREMENT TECHNIQUES

1. INTRODUCTION

The acoustic impedance of a material is its most basic acoustic property. Impedance, defined as the ratio of the pressure to the volume displacement at a given surface in a sound-transmitting medium [1], is usually a frequency-dependent complex number. Because the acoustic impedance of a material determines the acoustic response of the surrounding environs, careful impedance measurements are required to produce accurate system models. Acoustic enclosure designers can then use these models to select suitable materials for wall, floor, and ceiling coverings.

A number of acoustic impedance measurement techniques have been developed. Many of these techniques used an impedance tube and a single microphone [2] - [6] to determine the acoustic impedance based on the spatial location and magnitude of the maximum and minimum sound pressure levels at an acoustic resonance. (This type of measurement is sometimes referred to as ASTM Standard C 384 [7].) These methods have two disadvantages: the first is nonstationary instrumentation and the second is difficulty in measuring the measurement of maximum and minimum pressure levels in the impedance tube.

More recent acoustic impedance measurement methods have used two microphone systems [8] - [10] which require two similar, phase-calibrated microphones at some location in the tube with a known distance between them. The acoustic wave response is mathematically separated into its reflected and incident components using a transfer function between the microphones. This decomposition of wave propagation allows the computation of acoustic impedance. (This type of measurement is sometimes referred to as ASTM Standard E 1050 [11].) These measurements are sometimes difficult to calculate

because of numerical instability at certain frequencies and sensitivity to instrument calibration.

Another recent acoustic impedance measurement technique utilized the eigenvalues of the transfer function of the impedance tube to evaluate the impedance at the end of the tube [12]. One microphone recorded the input pressure at the end of the impedance tube while a second recorded the tube response at a different location. A fast Fourier transform of the system was calculated and its eigenvalues extracted. The functional form of the eigenvalues were then inverted, and the acoustic impedance at each duct resonance was computed. This method, while extremely stable, is a spectrally sparse identification technique; the impedance of the material at the end of the duct can only be computed at discrete duct resonances.

This report describes a study that compares two acoustic impedance measurement techniques: the wave decomposition technique and the inverse eigenvalue technique. A theoretical derivation is presented, and an experiment comparing acoustic impedance measurements from both methods is provided.

2. THEORY

2.1. WAVE DECOMPOSITION METHOD

The first method investigated is the wave decomposition method [8],[11]. The mathematics presented here are only slightly different than those from the reference [8],[11] derivation.

The system model is the wave equation designating a one-dimensional hard-walled duct. The model has a pressure boundary condition at one end that represents an excitation speaker and an impedance boundary condition at the other end that represents some material inserted in the end of the duct. This material produces a partially reflective and reactive boundary condition that results in a bounded complex transfer function of the duct.

The boundary condition at $x = L$ is the ratio between the pressure and the particle velocity and is expressed as [6],[13],[14]

$$\frac{\partial u}{\partial x}(L,t) = -K\left(\frac{1}{c}\right)\frac{\partial u}{\partial t}(L,t) \ , \tag{1}$$

where K is the complex acoustic impedance (dimensionless) at the end $x = L$, $u(L,t)$ is the fluid particle displacement (m) at $x = L$, c is the sonic wave speed in the duct (m/s), t is the time variable (s), x is the spatial variable (m), and L is the length of the duct (m). Acoustic impedance $K = 0 + 0i$ corresponds to a totally reflective open end, $|K| = \infty$ corresponds to a totally reflective closed end, and $K = 1 + 0i$ corresponds to a totally absorptive end where the dynamic response of the duct appears as if the duct were infinitely long. In general, the acoustic impedance is neither totally reflective nor absorptive, and K does not equal any of the above three values that correspond to ideal boundary conditions.

The homogeneous wave equation that models particle displacement in a linear, hard-walled one-dimensional duct can be expressed as [14],[15]

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0 \ , \tag{2}$$

where $u(x,t)$ is the particle displacement (m). The wave equation assumes an adiabatic system, uniform duct cross section and negligible air viscosity effects. All system dissipation is assumed to occur at the termination end $x = L$; in other words, the duct is acting as a pure energy transmitter. The one-dimensional assumption requires the diameter of the duct to be small compared with the wavelength of acoustic energy. This assumption is usually considered valid when $f < 0.586(c/d)$ where f is the frequency of the acoustic wave (Hz) and d is the diameter of the tube (m) [7],[11]. Although the forcing function of the excitation speaker could be added to the right-hand side of equation (2), it will be advantageous to include it as a boundary condition for this method.

The excitation speaker at the duct end at $x = 0$ is modeled as a harmonic pressure excitation (or source). This boundary condition is [16]

$$\frac{\partial u}{\partial x}(0,t) = \frac{-P_0}{\rho c^2} e^{i\omega t} \ , \tag{3}$$

where P_0 is the magnitude of the pressure source (N/m^2), ρ is the density of the medium (kg/m^3), ω is the frequency of the excitation (rad/s), and i is the square root of -1. Implicit in equation (3) is negligible source impedance. If the source impedance is not small, it can be incorporated into the model [17].

The acoustic pressure of the system is proportional to the spatial derivative of the particle displacement. This equation is [14]

$$P(x,t) = -\rho c^2 \frac{\partial u}{\partial x}(x,t) \ , \tag{4}$$

where $P(x,t)$ is the acoustic pressure response of the tube (N/m^2) at some location x.

Equations (1) - (4) can be solved by using separation of variables and then equating the ordinary differential equations to the boundary conditions. This technique results in the following response of the system [6],[16]:

$$\frac{P(x,t)}{P_0} = \left\{ \frac{(K-1)\exp\left[i\frac{\omega}{c}(x-L)\right] + (K+1)\exp\left[-i\frac{\omega}{c}(x-L)\right]}{(K-1)\exp\left(-i\frac{\omega}{c}L\right) + (K+1)\exp\left(i\frac{\omega}{c}L\right)} \right\} \exp(i\omega t) \ . \tag{5}$$

The transfer function $T(K,\omega,c,L)$ of the system is the coefficient of $\exp(i\omega t)$ in equation (5).

The acoustic impedance K may be determined by expressing $T(K,\omega,c,L)$ in an alternate form. This process begins by expressing the function $T(K,\omega,c,L)$ in terms of the real and imaginary parts as

$$\mathrm{Re}(T) + i\,\mathrm{Im}(T) = \frac{\mathrm{Re}(K)\cos\left[\frac{\omega}{c}(x-L)\right] + i\left\{\mathrm{Im}(K)\cos\left[\frac{\omega}{c}(x-L)\right] - \sin\left[\frac{\omega}{c}(x-L)\right]\right\}}{\mathrm{Re}(K)\cos\left(\frac{\omega}{c}L\right) + i\left\{\mathrm{Im}(K)\cos\left(\frac{\omega}{c}L\right) + \sin\left(\frac{\omega}{c}L\right)\right\}}, \tag{6}$$

where T is the transfer function data (dimensionless) determined experimentally. (Re( ) denotes the real value of a complex number, and Im( ) denotes the imaginary value of a complex number.) Although the transfer function data are normally displayed as a

magnitude and a phase angle, it is advantageous to leave them in the form of a real and an imaginary number for the computations discussed here. Equation (6) is now expressed as:

$$\begin{aligned}&\mathrm{Re}(K)\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right)-\mathrm{Im}(T)\mathrm{Im}(K)\cos\left(\frac{\omega}{c}L\right)-\mathrm{Im}(T)\sin\left(\frac{\omega}{c}L\right)\\&i\left[\mathrm{Im}(T)\mathrm{Re}(K)\cos\left(\frac{\omega}{c}L\right)+\mathrm{Re}(T)\mathrm{Im}(K)\cos\left(\frac{\omega}{c}L\right)+\mathrm{Re}(T)\sin\left(\frac{\omega}{c}L\right)\right]=\\&\mathrm{Re}(K)\cos\left[\frac{\omega}{c}(x-L)\right]+i\left\{\mathrm{Im}(K)\cos\left[\frac{\omega}{c}(x-L)\right]-\sin\left[\frac{\omega}{c}(x-L)\right]\right\}\quad.\end{aligned}\tag{7}$$

Equating the real and imaginary parts of equation (7) yields the system of equations given by

$$\begin{aligned}&\left\{\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right)-\cos\left[\frac{\omega}{c}(x-L)\right]\right\}\mathrm{Re}(K)-\mathrm{Im}(T)\cos\left(\frac{\omega}{c}L\right)\mathrm{Im}(K)\\&\qquad=\mathrm{Im}(T)\sin\left(\frac{\omega}{c}L\right)\end{aligned}\tag{8}$$

$$\begin{aligned}&\mathrm{Im}(T)\cos\left(\frac{\omega}{c}L\right)\mathrm{Re}(K)+\left\{\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right)-\cos\left[\frac{\omega}{c}(x-L)\right]\right\}\mathrm{Im}(K)\\&\qquad=-\sin\left[\frac{\omega}{c}(x-L)\right]-\mathrm{Re}(T)\sin\left(\frac{\omega}{c}L\right)\quad.\end{aligned}\tag{9}$$

Equations (8) and (9) can also be written in matrix form as

$$\mathbf{Ak}=\mathbf{b}\ ,\tag{10}$$

where

$$\mathbf{A}=\begin{bmatrix}\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right)-\cos\left[\frac{\omega}{c}(x-L)\right] & -\mathrm{Im}(T)\cos\left(\frac{\omega}{c}L\right)\\ \mathrm{Im}(T)\cos\left(\frac{\omega}{c}L\right) & \mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right)-\cos\left[\frac{\omega}{c}(x-L)\right]\end{bmatrix},$$

$$\mathbf{k}=\begin{bmatrix}\mathrm{Re}(K)\\ \mathrm{Im}(K)\end{bmatrix},\ \text{and}$$

$$\mathbf{b} = \begin{bmatrix} \mathrm{Im}(T)\sin\left(\dfrac{\omega}{c}L\right) \\ -\sin\left[\dfrac{\omega}{c}(x-L)\right] - \mathrm{Re}(T)\sin\left(\dfrac{\omega}{c}L\right) \end{bmatrix} .$$

When **A** is nonsingular, a unique solution for **k** is given by $\mathbf{k} = \mathbf{A}^{-1}\mathbf{b}$ such that

$$\begin{aligned} \mathrm{Re}(K) = \frac{1}{\Delta} \Bigg\langle & \left\{\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right) - \cos\left[\frac{\omega}{c}(x-L)\right]\right\}\mathrm{Im}(T)\sin\left(\frac{\omega}{c}L\right) \\ & + \mathrm{Im}(T)\cos\left(\frac{\omega}{c}L\right)\left\{\sin\left[\frac{\omega}{c}(x-L)\right] + \mathrm{Re}(T)\sin\left(\frac{\omega}{c}L\right)\right\} \Bigg\rangle \end{aligned} \tag{11}$$

and

$$\begin{aligned} \mathrm{Im}(K) = \frac{1}{\Delta} \Bigg\langle & \left\{\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right) - \cos\left[\frac{\omega}{c}(x-L)\right]\right\}\left\{-\sin\left[\frac{\omega}{c}(x-L)\right] - \mathrm{Re}(T)\sin\left(\frac{\omega}{c}L\right)\right\} \\ & - [\mathrm{Im}(T)]^2\cos\left(\frac{\omega}{c}L\right)\sin\left(\frac{\omega}{c}L\right) \Bigg\rangle \end{aligned} \tag{12}$$

where

$$\Delta = \det[\mathbf{A}] = \left\{\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right) - \cos\left[\frac{\omega}{c}(x-L)\right]\right\}^2 + \left\{\mathrm{Im}(T)\cos\left(\frac{\omega}{c}L\right)\right\}^2 . \tag{13}$$

Acoustic impedance K represents the acoustic impedance at the frequency ω; therefore at any frequency where the transfer function is measured, the impedance can be computed.

Because a determinant is involved in the calculations, there is some instability in the measurement technique. This instability occurs whenever the determinant of **A** is equal to (or near) zero. The determinant is strictly real-valued, therefore det(**A**)=0 when

$$\mathrm{Im}(T)\cos\left(\frac{\omega}{c}L\right) = 0 \tag{14}$$

and

$$\mathrm{Re}(T)\cos\left(\frac{\omega}{c}L\right) - \cos\left[\frac{\omega}{c}(x-L)\right] = 0 . \tag{15}$$

It is shown below that equations (14) and (15) are concurrently equal to zero when

$$\mathrm{Im}(T) = 0 . \tag{16}$$

The imaginary value of T can be rewritten from equation (6) as

$$\operatorname{Im}(T)=\frac{-\operatorname{Re}(K)\sin\left(\frac{\omega}{c}x\right)}{\left[\operatorname{Re}(K)\cos\left(\frac{\omega}{c}L\right)\right]^2+\left[\operatorname{Im}(K)\cos\left(\frac{\omega}{c}L\right)+\sin\left(\frac{\omega}{c}L\right)\right]^2} \quad . \tag{17}$$

Assuming the real part of the acoustic impedance is not zero, then equations (16) and (17) are satisfied when

$$\sin\left(\frac{\omega}{c}x\right)=0 \ . \tag{18}$$

Equation (18) implies that

$$\cos\left(\frac{\omega}{c}x\right)=\pm 1 \quad . \tag{19}$$

Inserting equations (18) and (19) into the real part of equation (6) yields

$$\operatorname{Re}(T)=\pm 1 \quad . \tag{20}$$

Inserting equations (18), (19), and (20) into the left-hand side of equation (15) enforces the equality. Therefore,

$$\operatorname{Im}(T)=0 \quad \Rightarrow \quad \det[\mathbf{A}]=0 \quad , \tag{21}$$

and impedance calculation instability occurs at

$$\omega=\frac{n\pi c}{x} \qquad n=0,1,2,\dots \ , \tag{22}$$

where x is the location of the response measurement microphone and ω is measurement frequency in rad/s.

Another possibility for the determinant of **A** to vanish is that

$$\cos\left(\frac{\omega}{c}L\right)=0 \ . \tag{23}$$

However, inserting equation (23) into equation (15) will produce a zero value (and consequently det(**A**)=0) only if $\sin\left(\frac{\omega}{c}x\right)=0$ which is identical to equation (18) and is

already considered above. Thus, equation (23) alone does not produce a zero valued determinant of **A**.

The system of equations can be evaluated when det(**A**)=0. If equations (16), (18), (19) and (20) are inserted into equation (10), the result is **A**=**b**=**0**. There are an infinite number of solutions for **k**, as any value will satisfy **Ak**=**b** when det(**A**)=0. Note that in order for an infinite number of solutions to exist, the vector **b** must be orthogonal to the null space of $\mathbf{A}^{\mathbf{T}}$. It may be shown that this requirement implies **b**=**0**, consistent with the above conclusion. If specific frequencies are of interest where impedance calculation instability occurs, the location of the response measurement microphone can be changed, and the impedance calculation instability will be moved to different frequencies.

2.2. INVERSE EIGENVALUE METHOD

The second method investigated is an inverse eigenvalue method [12]. The system model is similar to the previous one, except the forcing function has been moved from the boundary condition at $x = 0$ to the right-hand side of the wave equation. The termination end impedance is the same as above:

$$\frac{\partial u}{\partial x}(L,t) = -K\left(\frac{1}{c}\right)\frac{\partial u}{\partial t}(L,t) \ . \tag{24}$$

The linear second-order wave equation modeling particle displacement in a hard-walled one-dimensional duct is

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\delta(x)P_e(t)}{\rho} \ , \tag{25}$$

where $\delta(x)$ is the Dirac delta function and $P_e(t)$ is the excitation pressure of the speaker at $x = 0$. The duct end at $x = 0$ is now modeled as a totally reflective open end. This boundary condition is

$$\frac{\partial u}{\partial x}(0,t) = 0 \ . \tag{26}$$

Equation (26), along with the right-hand side of equation (25), models the speaker as a pressure source at $x = 0$.

The eigenvalues of the model are found by application of the separation of variables to equations (24) and (26) and the homogeneous version of equation (25). Separation of variables assumes that each term of the series solution is a product of a function in the spatial domain multiplied by a function in the time domain:

$$u(x,t) = X(x)T(t) \ . \tag{27}$$

Substituting equation (27) into the homogeneous version of equation (25) produces two independent ordinary differential equations, each with a complex-valued separation constant λ:

$$\frac{d^2X(x)}{dx^2} - \lambda^2 X(x) = 0 \tag{28}$$

and

$$\frac{d^2T(t)}{dt^2} - c^2\lambda^2 T(t) = 0 \ . \tag{29}$$

The separation constant $\lambda = 0$ is a special case where $X(x) = T(t) = 1$ to satisfy equations (24) and (26). Although $\lambda = 0$ is a separation constant of the system, it does not contribute to the pressure field in the duct and is therefore ignored for further computational purposes [18],[19]. The spatial ordinary differential equation (28) is solved for $\lambda \neq 0$ using the boundary condition in equation (26):

$$X(x) = e^{\lambda x} + e^{-\lambda x} \ . \tag{30}$$

The time-dependent ordinary differential equation yields the following general solution:

$$T(t) = Ae^{c\lambda t} + Be^{-c\lambda t} \ . \tag{31}$$

Applying the boundary condition in equation (24) to equations (29) and (30) yields $B = 0$ and the separation constant

$$\lambda_n = \frac{1}{2L}\log_e\left(\frac{1-K}{1+K}\right) - \frac{n\pi i}{L} \quad , \quad n = 0, \pm 1, \pm 2, \ldots \ . \tag{32}$$

The system eigenvalues Λ_n are equal to the separation constant multiplied by the wave speed c ($\Lambda_n = c\lambda_n$). Each of these eigenvalues are functions of acoustic impedance, K. The inverse function will allow impedance K to be computed from the measured eigenvalues.

The acoustic impedance K of the end can now be determined at each duct resonance from the eigenvalue at that resonance. This computation assumes that the eigenvalues of the system are known. Measurement of these duct system eigenvalues is discussed in the next section. Directly solving for K in terms of Λ is very difficult; therefore, an intermediate variable β is introduced to simplify the acoustic impedance computation. The variable β_n is related to the nth eigenvalue Λ_n from equation (32) as

$$\mathrm{Re}(\Lambda_n) + i\,\mathrm{Im}(\Lambda_n) = \frac{c}{2L}\log_e[\mathrm{Re}(\beta_n) + i\,\mathrm{Im}(\beta_n)] - \frac{n\pi c i}{L}\,, \tag{33}$$

where the subscript "n" denotes the nth term. Equation (33) is now broken into two parts, one equating the real coefficients and the other equating the imaginary coefficients. The complex logarithm on the right-hand side is rewritten as

$$\log_e[\mathrm{Re}(\beta_n) + i\,\mathrm{Im}(\beta_n)] = \log_e|\beta_n| + i\arg(\beta_n) \tag{34}$$

where $|\beta_n|$ is the magnitude of β_n and $\arg(\beta_n)$ is the argument of β_n (i.e., the arctangent of $[\mathrm{Im}(\beta_n)/\mathrm{Re}(\beta_n)]$.)

The intermediate variable β_n is now solved in terms of the real and imaginary parts of the eigenvalues. The real part of β_n is

$$\mathrm{Re}(\beta_n) = \pm\left[\frac{\exp\left(\dfrac{4L\,\mathrm{Re}(\Lambda_n)}{c}\right)}{1+\tan^2\left(\dfrac{2Ld_n}{c}\right)}\right]^{1/2}, \tag{35a}$$

where

$$d_n = \mathrm{Im}(\Lambda_n) - \frac{nc\pi}{L}\,.$$

The sign of $\mathrm{Re}(\beta_n)$ in equation (35a) is determined by

$$\operatorname{sgn}[\operatorname{Re}(\beta_n)] = \begin{cases} +1 & \text{if } 0 \le |\Delta| \le 0.25 \\ -1 & \text{if } 0.25 \le |\Delta| \le 0.50 \end{cases}, \tag{35b}$$

where

$$\Delta = \frac{d_n}{\left(\frac{c\pi}{L}\right)} .$$

If the value of Δ is less than -0.5 or greater than 0.5, the eigenvalue index n is incorrect and corresponds to an eigenvalue other than the nth one. The value, n, must then be changed to produce a Δ between -0.5 and 0.5 that will correspond to the correct eigenvalue index. Once $\operatorname{Re}(\beta_n)$ is found, $\operatorname{Im}(\beta_n)$ is obtained by the equation

$$\operatorname{Im}(\beta_n) = \operatorname{Re}(\beta_n)\tan\left(\frac{2Ld_n}{c}\right), \tag{36}$$

where $\operatorname{Re}(\beta_n)$ is given in equation (35).

The term $(1\text{-}K)/(1\text{+}K)$ is now equated to the intermediate variable β_n using equations (32) and (33) as

$$\operatorname{Re}(\beta_n) + i\operatorname{Im}(\beta_n) = \frac{1 - \operatorname{Re}(K_n) - i\operatorname{Im}(K_n)}{1 + \operatorname{Re}(K_n) + i\operatorname{Im}(K_n)}, \tag{37}$$

where $\operatorname{Re}(K_n)$ is the real part of K and $\operatorname{Im}(K_n)$ is the imaginary part of K for the nth eigenvalue. Breaking equation (37) into two equations and solving for K_n as a function of β_n yields the acoustic impedance as

$$\operatorname{Re}(K_n) = \frac{1 - [\operatorname{Re}(\beta_n)]^2 - [\operatorname{Im}(\beta_n)]^2}{[\operatorname{Re}(\beta_n) + 1]^2 + [\operatorname{Im}(\beta_n)]^2} \tag{38}$$

$$\operatorname{Im}(K_n) = \frac{-2\operatorname{Im}(\beta_n)}{[\operatorname{Re}(\beta_n) + 1]^2 + [\operatorname{Im}(\beta_n)]^2} . \tag{39}$$

Acoustic impedance measurement K_n represents the acoustic impedance at the nth resonant frequency.

3. EXPERIMENT

The two acoustic impedance measurement techniques were compared experimentally. A 2.94-m long, 0.0762-m (3-in.) diameter PVC schedule 40 duct with a speaker mounted at one end was fabricated. A piece of packing foam was inserted at the termination end. This foam provided a boundary condition that was neither highly reflective nor highly absorptive. Speaker input pressure was measured at the entrance to the duct with a Bruel and Kjaer Type 4166 half-inch microphone attached to a Hewlett-Packard 5423A digital signal analyzer. The response of the duct was measured at x = 0.792 m with a second Bruel and Kjaer Type 4166 half-inch microphone, also attached to the digital signal analyzer (figure 1). The microphones were phase matched and calibrated with a Bruel and Kjaer Type 4230 sound level calibrator. Although a digital signal analyzer was used in this experiment, the same measurements could be made with a gain-phase meter. The digital signal analyzer is more useful for the inverse eigenvalue method because it contains an algorithm to extract the complex-valued eigenvalues from the transfer function of the system. This algorithm is transparent to the user.

The measured frequency response function is shown in figures 2 and 3. Figure 2 is the function illustrated using real and imaginary parts. Figure 3 is the function displayed as magnitude and phase angle, which is the conventional form in which transfer functions are displayed. The data Re(T) and Im(T) used in the first measurement technique are taken directly from figure 2. These data provide two of the parameters used to determine the acoustic impedance. The other parameters required are the speed of sound in the duct, the length of the duct, and the measurement frequency. All the parameters are relatively easy to measure. The measured duct eigenvalues, listed in table 1, were extracted from five independent transfer functions with the response measurement microphone at five different locations from x = 0.792 m through x = 1.42 m at 0.157 m increments. These eigenvalues are used in the second measurement technique. Note the low value of the standard

deviations in columns three and five of table 1. Table 2 shows the calculated acoustic impedance measurements for this second method.

The computed acoustic impedance from both methods is displayed graphically in figure 4. The acoustic impedances using the wave decomposition theory [8] are displayed as x's and +'s. The real value of K is shown on the upper plot as x's and the imaginary value of K is shown on the lower plot as +'s. The acoustic impedance measured using the eigenvalue inverse technique [12] is shown on both plots as squares. There is an extremely close agreement for the real value of K and relatively close agreement for the imaginary value of K. For this specific case, the acoustic impedance is dominated by the real value of K, and the small difference in the imaginary values of K is negligible. The divergence of the calculated acoustic impedance around 217 Hz for the wave decomposition theory can be viewed clearly. This divergence is predicted in the previous section by the theory.

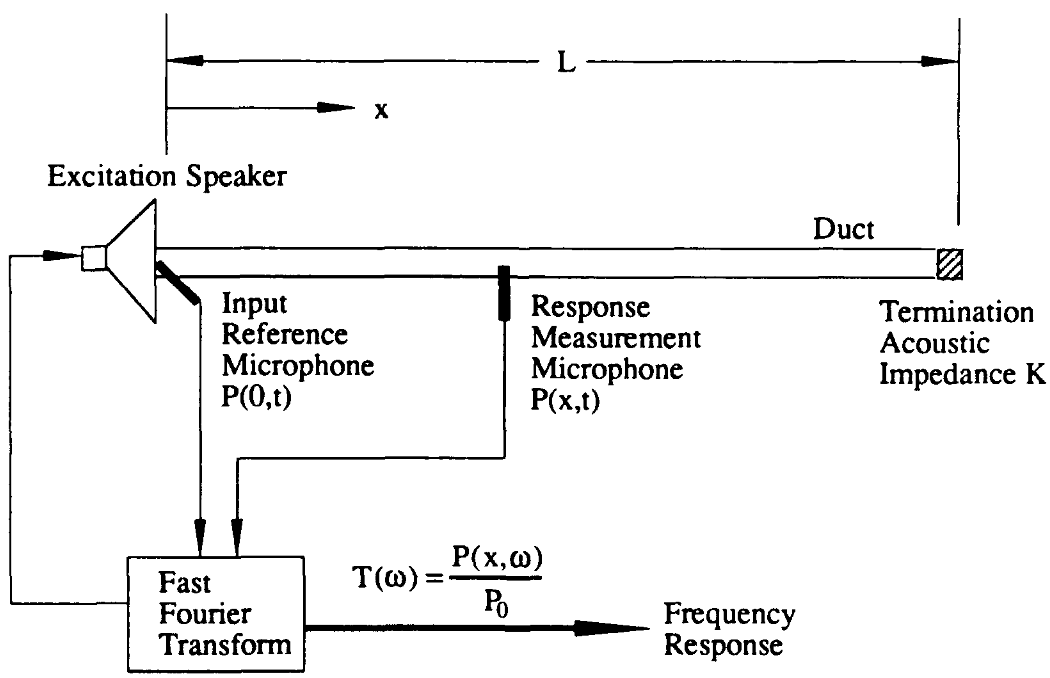

Figure 1. Laboratory Configuration

Table 1. Measured Duct Eigenvalues for the Impedance Tube

Eigenvalue (n)	$Re(\Lambda_n)$ mean, m	$Re(\Lambda_n)$ std. dev., s	$Im(\Lambda_n)$ mean, m (Hz)	$Im(\Lambda_n)$ std. dev., s (Hz)
1	-4.955	0.033	57.89	0.045
2	-5.204	0.124	116.3	0.055
3	-5.713	0.091	174.7	0.152
4	-5.755	0.131	233.8	0.259
5	-4.231	0.122	291.7	0.164
6	-5.414	0.117	350.5	0.192

Table 2. Calculated Acoustic Impedance Using Inverse Eigenvalue Method

Eigenvalue (n)	$Re(K_n)$	$Im(K_n)$
1	0.260	0.032
2	0.273	0.037
3	0.298	0.042
4	0.300	0.014
5	0.224	0.046
6	0.283	0.031

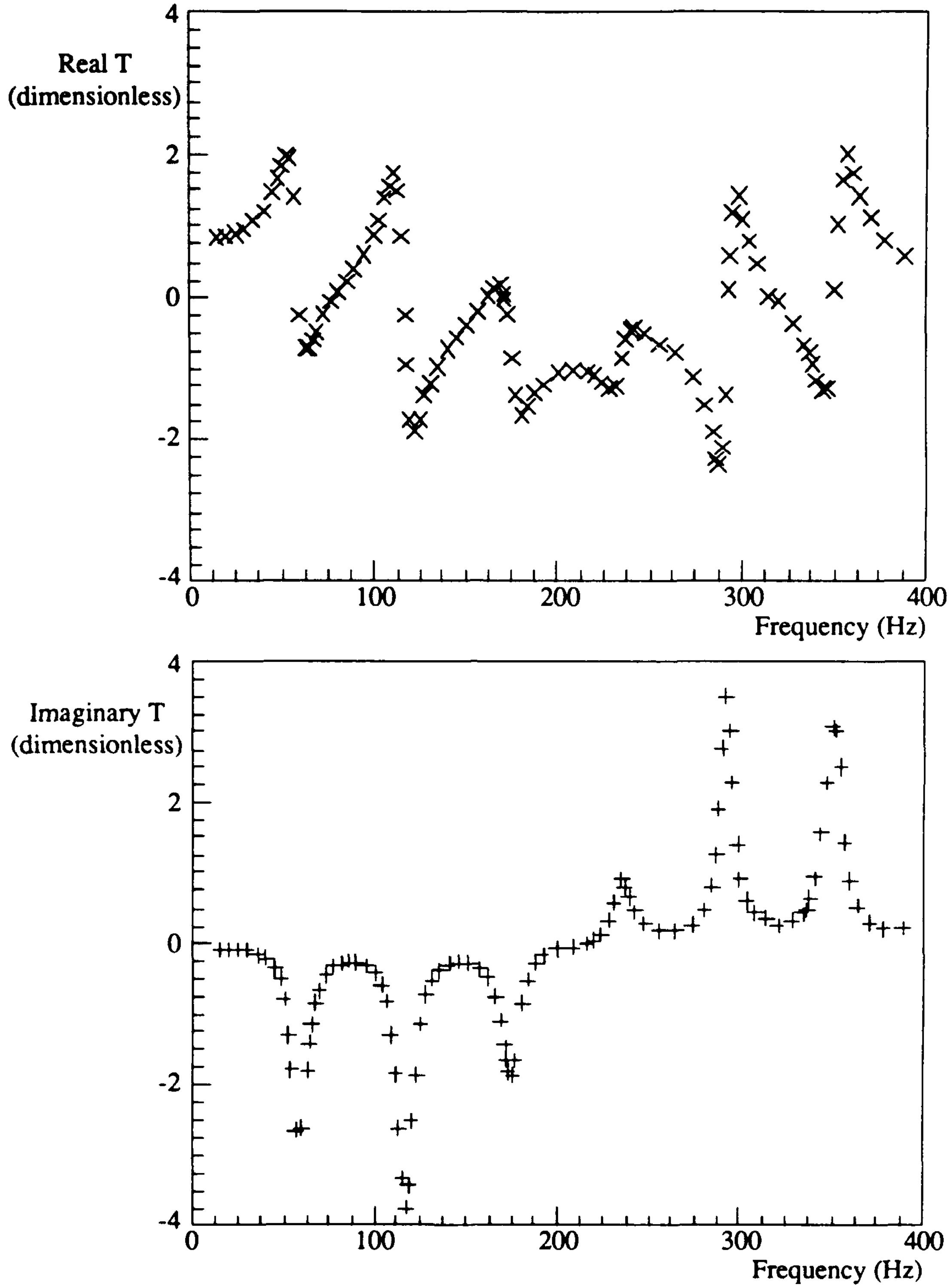

Figure 2. Transfer Function Real and Imaginary Terms Versus Frequency

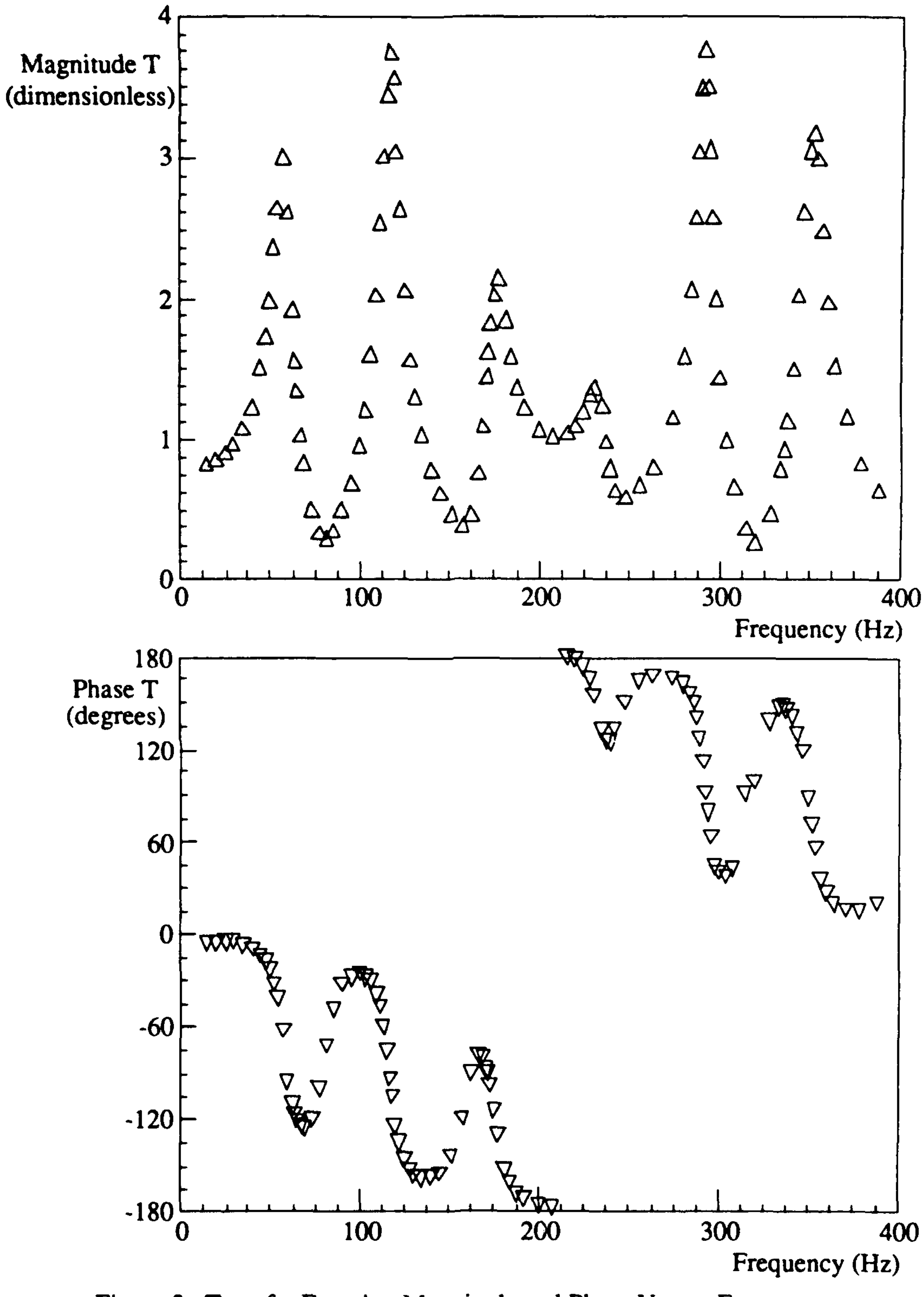

Figure 3. Transfer Function Magnitude and Phase Versus Frequency

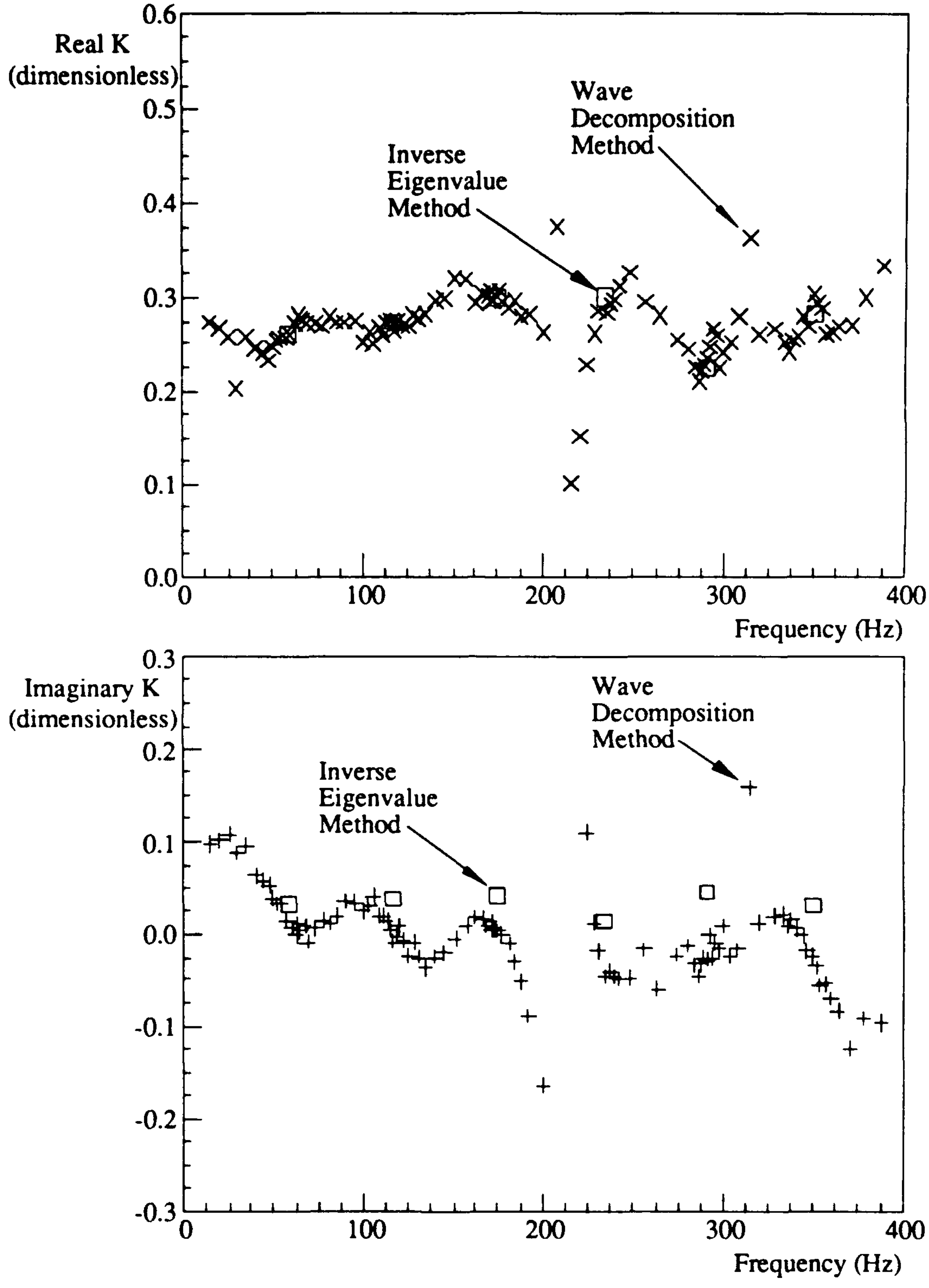

Figure 4. Acoustic Impedance Versus Frequency

4. CONCLUSIONS

Both the wave decomposition and inverse eigenvalue measurement techniques provided similar values of acoustic impedance for the material tested. The wave decomposition method is useful because it provides a measure of impedance at every frequency where the transfer function is measured. The inherent disadvantage is its numerical instability at or near certain frequencies. Although the inverse eigenvalue method is an extremely numerically stable measurement technique it provides a measure of acoustic impedance only at duct resonances. Other values must be interpolated.

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